Comment on “Uncertainty in measurements of distance”

Y. Jack Ng and H. van Dam
Institute of Field Physics, Department of Physics and Astronomy,
University of North Carolina, Chapel Hill, NC 27599
yjng@physics.unc.edu

ABSTRACT

We have argued that quantum mechanics and general relativity give a lower bound $\delta l \gtrsim l^{1/3} l_P^{2/3}$ on the measurement uncertainty of any distance l much greater than the Planck length l_P. Recently Baez and Olson have claimed that one can go below this bound by attaching the measuring device to a massive elastic rod. Here we refute their claim. We also reiterate (and invite our critics to ponder on) the intimate relationship and consistency between black hole physics (including the holographic principle) and our bound on distance measurements.

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We begin by recapitulating our results on distance measurements. (Ng & van Dam 1994, 1995) Our measuring device consists of a clock (which also serves as a light-emitter and receiver) of mass m and a mirror, placed respectively at the two points the distance between which we want to measure. By sending a light signal from the clock to the mirror in a timing experiment, we can determine the distance l. Following Wigner(Wigner 1957; Salecker & Wigner 1958) we argue that quantum mechanics implies an uncertainty in the distance measurement given by

$$\delta l \gtrsim \left(\frac{\hbar l}{mc}\right)^{1/2}. \tag{1}$$

We also argue that general relativity can be used to yield

$$\delta l \gtrsim \frac{Gm}{c^2}. \tag{2}$$

Squaring the uncertainty from Eq. (1) and multiplying the result by Eq. (2), we obtain(Ng & van Dam 1994, 1995)

$$\delta l \gtrsim (l l_P^2)^{1/3}, \tag{3}$$

where $l_P \equiv (\hbar G/c^3)^{1/2}$ is the Planck length.[1]

[1]See also the discussions in Ref.(Karolyhazy 1966) and Ref.(Sasakura 1999).

Recently Baez and Olson(Baez & Olson 2002) have argued that one can go below the bound given above by attaching the clock (of negligible mass) to one end of a massive rod. As in our own gedanken experiment, a distance l is measured by sending a burst of light from the clock and measuring the time it takes for the light to return. According to Baez and Olson, the uncertainty δl receives contributions from two sources. They argue that the uncertainty of the clock's position with respect to the rod's center of mass contributes an amount $(L\hbar/Mc)^{1/2}$, where L is the equilibrium length of the rod and M is its mass. They also estimate that the uncertainty of the position of the rod's center of mass contributes an amount $(l\hbar/Mc)^{1/2}$. Assuming that the two sources of uncertainties are uncorrelated and arguing that the rod must be longer than its Schwarzschild radius, i.e., $L \gtrsim GM/c^2$, they obtain

$$\delta l \gtrsim \left(\sqrt{\frac{GM}{c^2}} + \sqrt{l}\right)\sqrt{\frac{\hbar}{Mc}}. \tag{4}$$

By using a very heavy rod so that $l \lesssim GM/c^2$ they

conclude that

$$\delta l \gtrsim l_P. \qquad (5)$$

Without commenting on the details of their argument we merely note that their conclusion Eq. (5) depends on the inequalities

$$L \gtrsim GM/c^2 \gtrsim l. \qquad (6)$$

While one cannot expect a measuring device to be able to measure an abitrarily small distance, one does think a good device should be able to measure any distance bigger than a certain minimum length. If so, then it follows that Baez and Olson's measuring device has to be very large and very massive. We do not believe it can serve as an ideal clock to uncover fundamental properties of spacetime. The use of a huge and massive clock will completely overwhelm the minute uncertainty in distance measurements. We also observe (Ng & van Dam 2000a) that one can actually obtain Baez and Olson's result Eq. (5) by using our own measuring device and following our own argument if, for the bound on the mass m of the clock, instead of Eq. (2), one uses

$$l \gtrsim Gm/c^2, \qquad (7)$$

which is nothing but the mathematical statement of the obvious fact that, to measure the distance from the clock to the mirror, the mirror should not be inside the Schwarzschild radius of the clock. Now let us recall that Baez and Olson's measuring device has a clock attached to a massive rod. Thus one can regard the mass M of the rod to be the effective mass of the whole measuring device which serves as a clock. It is curious that, to arrive at Eq. (5), one needs the distance l to be smaller than the Schwarzshild size of the measuring device according to Baez and Olson, whereas the opposite is true according to us. Alternatively one can interpret Baez and Olson's disagreement with us as arising from disagreement on whether the more restrictive bound on m given by Eq. (2) (as compared to the bound given by Eq. (7)) is also correct. We think so and have commented on this issue in Ref.(Ng & van Dam 2000a).

We do not claim that our measuring device is ideal and it is probably impossible to give a theoretical proof that our argument is sound beyond doubt. In lieu of such a proof let us recall some pieces of plausible "circumstantial" evidence in support of our claim. First of all, our bound on the uncertainty of distance measurements appears to be consistent(Ng & van Dam 2000a,b; Ng 2001) with the holographic principle('t Hooft 1993; Susskind 1995) which states that the maximum number of degrees of freedom that can be put into a region of space is given by the area of the region in Planck units. To see this, let us consider a region measuring $l \times l \times l$. According to conventional wisdom (Eq. (5)), the region can be partitioned into cubes as small as l_P^3. It follows that the number of degrees of freedom of the region is bounded by $(l/l_P)^3$, i.e., the volume of the region in Planck units, contradicting the holographic principle. But according to our bound Eq. (3), the smallest cubes inside that region have a linear dimension of order $(ll_P^2)^{1/3}$. Accordingly, the number of degrees of freedom of the region is bounded by $[l/(ll_P^2)^{1/3}]^3$, i.e., the area of the region in Planck units, as required by the holographic principle.

It is interesting that an argument, very similar to that used by us to derive the lower bound on the uncertainty of distance meaurements, can be applied to relate the precision of any clock to its lifetime. (Ng 2001, 2002) For a clock of mass m, if the smallest time interval that it is capable of resolving is t and its total running time is T, one finds

$$t \gtrsim \left(\frac{\hbar T}{mc^2}\right)^{1/2}, \qquad (8)$$

the analogue of Eq. (1), and

$$t \gtrsim \frac{Gm}{c^3}, \qquad (9)$$

the analogue of Eq. (2). Let us now apply these two (in-)equalities to a black hole (of mass m) used as a clock. It is reasonable to use the light travel time across the black hole's horizon as the resolution time(Barrow 96; Ng 2001) of the clock,[2] i.e., $t \sim \frac{Gm}{c^3}$, then using Eq. (8) and Eq. (9), one immediately finds that

$$T \sim \frac{G^2 m^3}{\hbar c^4}, \qquad (10)$$

[2] We should rebut a possible objection. One might think that, due to Hawking radiation, the light signal cannot return to the point it started, thereby making our "experimental" arrangement impossible. But any realistic photon detector has a finite size and it can be moved slightly in anticipation of the return of the photon signal to detect the photon.

which is Hawking's black hole lifetime! Thus, if we had not known of black hole evaporation, this remarkable result would have implied that there is a maximum lifetime (of this magnitude) for a black hole. This is another "circumstantial" evidence in support of our bound Eq. (3) (actually, also separately, of Eq. (1) and Eq. (2)).

There is yet another piece of "circumstantial" evidence. It is related to black hole entropies and the ultimate physical limits to computation. But the evidence is more indirect. Interested readers are referred to Ref.(Ng 2002).

Baez and Olson are not our only critics. Some critics claim that Eq. (1) is wrong,(Adler et al. 2000; Ozawa 2002) and some are sure that the use of Eq. (2) is unwarranted, though the above "derivation" of Hawking's black hole lifetime seems to indicate that there is some validity to the analogues of the two equations. Some critics tend to think that the relationship between the bound on length uncertainty and the holographic principle has not been satisfactorily proven. Some doubt that the above "derivation" of Hawking's black hole lifetime is anything but a simple exercise in dimensional analysis. Conveniently they ignore the fact that there are more than one dimensionful quantity in the problem. Still some believe that metric fluctuations corresponding to Eq. (3) yield an unacceptably large fluctuations in energy density.(Diosi & Lukacs 1996) But actually the energy density is extremely small(Ng & van Dam 1997) and is of the same order of magnitude as that for the metric fluctuations corresponding to Eq. (5); spacetime fluctuations hardly cost any energy!

Conventional wisdom says that gravitational effects are important only at distances comparable to the Planck scale. Any proposals suggesting that the uncertainty in length is not the Planck length l_P should be closely scrutinized. We think proponents of such proposals should address the following questions:
1. Does the proposal contradict logic or experimental facts?
2. Are there hints that such a proposal deserves looking into?
3. What are the consequences of the proposal?
We have applied the above three criteria to our own proposal:
1. To the best of our knowledge, our proposal does not contradict logic or experimental facts.
2. That the uncertainty in length depends on more than one length scale (as given by Eq. (3)) is not surprising if we recall(Ng & van Dam 1995) that the uncertainty in length of a thin long ruler also depends on more than one length scale, viz., the length of the ruler itself as well as the lattice spacing and the thermal wavelength at low and high temperature respectively.
3. Our proposal is consistent with (semi-classical) black hole physics. And the surprisingly large uncertainties arising from distance measurements according to our proposal may one day be detectable with improved modern gravitational-wave interferometers(Amelino-Camelia 1999; Ng & van Dam 2000b) or with improved modern laser-based atom interferometers(Ng 2002). Our work(Ng & van Dam 1997, 2000a) also indicates that weak gravitational waves, as needed to provide our lower bound on uncertainty in length, do not have the energy to deform rulers or bars (as required for a positive signal of gravitational wave in the Weber aluminum bar experiment), but they do deform spacetime enough to produce the $(ll_P^2)^{1/3}$ result.
Our proposal adequately satisfies the three criteria.

Our arguement may not be air-tight, but we do not think that Baez and Olson(Baez & Olson 2002) have disproved our claim.

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